
AndrewJeffrey
presents
ALWAYS
SometimeS
Never
?

This book is dedicated to everyone who has ever changed my thinking. Thank you for not letting me stay as I was.

To those who have ever been challenged by mathematics; fear not, for this is as it should be. Challenge strengthens us, so let us

ALWAYS be prepared to admit that

SOMETIMES we don't know the answer, and

NEVER give up trying.

After all, it is through our persistence that we will succeed.

AJ, 2012

Front Cover design by the very talented Niamh McHale, age 10.

ALWAYS, SOMETIMES, NEVER ?

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The Cards

Introduction

Every so often an idea comes along that is a valuable addition to the bank of resources that exist to support the teaching and learning of mathematics.

The first time I ever saw a set of ***Always, Sometimes, Never*** cards, I knew that this was just such an idea. I started to look around for sets of these to use, and was surprised to discover that nobody had ever created a book (*or better still, an electronic book*) containing sets of cards for several different areas of the primary mathematics curriculum.

So here it is. After months of research, I have put together what I believe may be the first complete book of ***Always, Sometimes, Never*** resources available.

To make ***Always, Sometimes, Never*** as user-friendly and cost-effective as possible, not only do you receive the book in .pdf format, but also a license to make unlimited photocopies of any of the cards for use *in your own school* or institution.

I hope you will enjoy using these as much as I have enjoyed creating them, and I am confident that your children will look forward to playing ***Always, Sometimes, Never*** with enthusiasm!

Andrew Jeffrey, 2012

Getting to know the *Always, Sometimes, Never* cards

The cards are arranged in sets of 6. Each card bears a statement based on an area of the mathematics curriculum. Sometimes there is more than one set of cards for each topic, broadly differentiated into levels of difficulty. This allows you to cater more easily for a range of abilities within your class. Of course there is nothing to stop you combining sets of cards into one large set if you wish! Sets are graded as **L** (Lower) **M** (Middle) and **H** (Higher). Please note that this is simply a rough indication of *relative* difficulty; it should not be assumed that children working at similar National Curriculum levels will find the same things easy of course, as it depends entirely on their particular understanding of a particular point, and on any misconceptions they may have picked up.

The main aim of the activity is for children to decide whether each statement is **always**, **sometimes** or **never** true, and to sort the statements into groups.

Here is an example of each type:

This is **sometimes** true. Some do, some don't. For example, a rectangle and an isosceles trapezium.

Quadrilaterals have right angles

This is **always** true - a square is a special type of rectangle that just happens to have all the sides the same length.

Squares are rectangles

Interestingly, this is **never** true. This can be demonstrated by squaring the numbers from 1-10.

Square numbers end in an '8'

Most sets of cards will have examples of all three categories but this should NOT be assumed – this means that children cannot reason (spuriously) that '*we have loads of '**sometimes**' and '**always**' so this last one must be a '**never**'.* The right-hand pages contain the answers, together with a brief reason.

Perhaps the *real* value of these activities comes from the reasoning and communication that is necessary in order to complete the task successfully, so it is recommended that this activity is <u>not</u> seen as a task to be tackled individually in silence.

This is not to say it cannot be done by individuals FIRST - they could all have a sheet, note down their own thoughts, then collate and try to converge on an agreed answer. This is a very strong process to support learning, and the next page gives details of several more powerful ways to use this activity.

I think highly enough of this to use it on virtually every training day I offer for schools, and there aren't many things about which I can make such a claim!

Ambiguity

In some cases, the answer will depend on user-definitions; for example 'numbers' might be taken to mean 'whole' or 'positive'. Where there is any ambiguity there is value in deciding with the group beforehand which definition should be used.

As a general guiding principle, I suggest that we contain ourselves to REAL numbers rather than IMAGINARY ones, in which case (for example) a card with 'square numbers are negative' could safely be put into 'NEVER'.

The case of zero as a real number comes up several times - be warned!

Working with A-level students of course we would dispense with this principle**!**

Let's get stuck in…

In the classroom: how to use *Always, Sometimes, Never*

One of the great things about ***Always Sometimes Never*** is the flexibility it offers. Here are a few effective ways to do so:

a) **Whole class (1):** Print out large versions of the cards to hold up, or display them on an IWB. For each statement, discuss it as a class, then decide in which category the statement belongs.

b) **Whole Class (2):** Display the cards as in (1), but this time children work in pairs or small groups to decide on their answer. Each pair or group should then vote by holding up a pre-printed card or whiteboard showing a large A, S or N card.

c) **Whole Class (3):** The statements are printed out onto A4 cards and each group is given a card. Three areas of the classroom are allocated to represent 'Always' Sometimes' and 'Never' and children should decide where to stand. To avoid congestion, only one person from each group needs to take the card to their chosen location. More than one group of children can have the same statement, a leading to some high-quality debates. It is sometimes interesting to give ALL the children the same statement!

d) **Pairs :** Give each pair of children a set of cards. They must discuss the statements and sort the cards into three piles. Once each pair has finished, they can discuss their answers with another pair sitting nearby. They can change their answers but only if they can give a convincing reason why they have done so.

e) **Groups:** Distribute a set of cards to a group of children – they must read out their card to the rest of their group, and the group must then decide in which of the three categories it belongs. It can only be placed once EVERYONE agrees.

f) **As a summary** to assess learning. For example, if you have been working on percentages, you might want to generate a set or statements that refer specifically to that topic and ask children to sort and discuss the statements.

g) **Independently**: Cut out the statements and stick their solution card (on the facing page) to the back. The cards can then be used independently as a pack of cards by individual children, as they are self-correcting.

However you use the cards, the key element in all the activities is the REASONING that children must do, both internally and aloud, as they seek to justify their decisions.

These are just a few of the many ways to use the cards, but you may well think of more (*post-its? working walls?)* as you become more familiar with the cards. Enjoy!

Odd and Even (1)

L

Even numbers end in zero	Whole numbers ending in zero are even
Odd numbers end in 1,3,5,7 or 9	Half an odd number is a whole number
Half an even number is a whole number	Twice a whole number is even

Odd and Even (1)

L

SOMETIMES 2,4,6,8 don't, but 10 does. 12,14, 16, 18 don't, but 20 does. And so on.	ALWAYS They are all multiples of 10, which is in the 2x table, and therefore all even.
ALWAYS 1,3,5,7,9,11,13,15,17,19,21,23 etc. All end in one of the odd digits.	NEVER Odd numbers don't divide by 2 so half of them will never be a whole number.
ALWAYS Any even number is 2 times a whole number. Halving gets us back to that number.	ALWAYS Twice a number means 'the number times 2', so will always be in the 2x table (or even)

Odd and Even (2)

M

The sum of an odd and an even number is odd	Half an even number is an odd number
Even numbers contain the digit 3	The sum of two even numbers is odd
Even numbers are bigger than odd numbers	Half an odd number is an odd number

Odd and Even (2)

M

ALWAYS **An odd number is an even number plus. Adding this to another even will give a larger even number plus one – an odd number.**	**SOMETIMES** **Half of 12 is 6** **Half of 14 is 7.**
SOMETIMES **24 doesn't** **34 does.**	**NEVER** **Adding two multiples of 2 will give another multiple of 2.**
SOMETIMES **2 is bigger than 1 but 2 is smaller than 3.**	**NEVER** **Halving any odd number will not give us a whole number as odd numbers are defined as numbers that DONT divide by 2.**

Odd and Even (3)

M

Numbers that end in a ‘3’ are odd	Numbers ending in a ‘4’ are even
Even numbers end in a ‘2’	Numbers ending in a ‘6’ are odd
The sum of two odd numbers is even	Twice a whole number is even

Odd and Even (3)

M

ALWAYS As are numbers that end in 1,5,7 or 9	ALWAYS As are numbers that end in 2,6,8, or 0
SOMETIMES They could also end in 4,6,8, or 0	NEVER They are always even
ALWAYS Each odd number is an even number plus one. Take the two ‘plus ones’ together to make 2: another even!	ALWAYS ‘Twice’ is the same as ‘two times’ so it will be in the 2x table.

DICE

(assume regular 6-sided)

M

If I throw a dice I will get a 7	If I throw a dice I will get a 6
If I throw 2 dice and add them the most likely total is 7	If I throw a dice I will get a whole number
6 comes up less often than other numbers on a dice	if I throw a dice four times I will get 2 odd and 2 even numbers

DICE

(assume regular 6-sided)

M

NEVER You can only get the numbers 1,2,3,4,5 and 6	**SOMETIMES** All numbers are equally likely so you will get a 6 about ⅙ of the time. The more throws you do, the closer this amount it is likely to be to ⅙.
ALWAYS There are more ways to get 7 than any other total so it is ALWAYS more likely, though this doesn't mean it will always happen!	**ALWAYS** 1,2,3,4,5,6 are the only possibilities and are all whole
SOMETIMES Each number is equally likely to occur, but if you roll a dice lots of times you might get more 6s than 2s (or more 3s or more 5s)	**SOMETIMES** But you might for example get all sixes!

Properties of Polygons (1)

L

Triangles have exactly 2 lines of symmetry	Squares are rectangles
A polygon has 3 or more sides	Polygons have curved sides
A polygon has fewer than 1 million sides	Regular polygons have an even number of sides

Properties of Polygons (1)

L

NEVER **Scalene have none, isosceles have 1, and equilateral have 3. There are <u>no</u> triangles with 2 lines of symmetry.**	**ALWAYS** **Squares are special types of rectangles which have all the sides the same length.**
ALWAYS **The fewest number of sides on a polygon is 3 (triangle)**	**NEVER** **To be a polygon, a shape MUST have straight sides which meet at the ends but do not cross, and which contain a single area.**
SOMETIMES **There is no upper limit to the number of sides that a polygon may have.**	**SOMETIMES** **Equilateral triangles have 3, for example.**

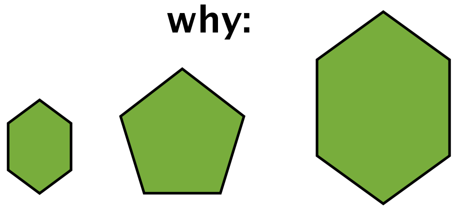
Properties of Polygons (2)

M

Identical quadrilaterals tesselate	Hexagons are larger than pentagons
Triangles have more than one obtuse angle	A Polygon has the same number of sides as diagonals
rectangles are squares	Rectangles have exactly 2 lines of symmetry

Properties of Polygons (2)

M

ALWAYS As long as you have a set of identical quadrilaterals it is always possible to arrange them with no gaps. Try it!	**SOMETIMES** Look at these shapes to see why: 
NEVER Since this would give a total of more than 180 degrees which is impossible.	**SOMETIMES** A pentagon is the only polygon that has this property; triangles and quadrilaterals have fewer diagonals, all others have more.
SOMETIMES A Square is a special sort of rectangle that has all its sides equal in length.	**SOMETIMES** Square ones have 4 lines!

Factors and Multiples (1)

M

A whole number greater than 1 has at least 2 factors	Even numbers contain the digit 3
Multiples of 5 end in a 5	Two multiples of 5 add up to a multiple of 10
Square numbers have an odd number of factors	Numbers ending in 3 are in the 3 times table

Factors and Multiples (1)

M

ALWAYS 1 and itself (and others unless it is prime.	SOMETIMES 32 contains a 3 but 24 does not yet both are even numbers.
SOMETIMES 5 does, 10 doesn't, etc.	SOMETIMES Works with 15+5 but not 20+5
ALWAYS Factors are in pairs except the square root which is in a pair with itself.	SOMETIMES 23 isn't but 33 is.

Factors and Multiples (2)

M

Doubling a number gives a higher number	Even numbers in the 3 times table have a factor of 6
Multiplying makes numbers bigger	If a number is a factor of 20 it is also a factor of 40
The sum of 3 consecutive whole numbers is a multiple of 3	Adding a zero to the end makes a number 10 times larger

Factors and Multiples (2)

L

SOMETIMES Doubling a negative numbers gives a lower number but doubling a positive number gives a higher one. Doubling zero does nothing!	ALWAYS The even multiples of 3 are 6,12,18, etc.
SOMETIMES But multiplying by some fractions or negatives can make numbers smaller.	ALWAYS Because 20 is itself a factor of 40
ALWAYS Try a few numbers to see this works; you could use cubes to figure out why.	SOMETIMES It only works for whole numbers, not fractions or decimals

Percentages

H

Finding 10% is the same as dividing by 10	Finding 20% is the same as dividing by 20
Increasing something by 10% then reducing it by 10% gets back to the original number	50% of an amount is the same as halving it
Reducing something by 30% is the same as multiplying it by 0.7	You can have more than 100% of something

Percentages

H

ALWAYS 10% means 10 out of 100 which is a tenth, hence it is the same as dividing by ten.	NEVER 20% means 20 out of 100 which is a fifth, hence it means dividing by 5 – not by 20.
NEVER It actually gets back to 99% of the original number, because 1.1 x 0.9 = 0.99	ALWAYS 50% means half
ALWAYS As there will be 70% of it left, and this is the same as 0.7	SOMETIMES If it is a number you can. If it is a finite thing such as a particular person you can't.

Averages

M

The mean of two numbers is halfway between them	The mean of 3 numbers is the middle one when they are in order
The median of 4 number is a whole number	The mean and the mode are the same number
The median of a set of numbers is larger than the mode	The mean of a list of numbers is larger than any number in the list

Averages

M

ALWAYS Because this is exactly how you calculate the mean of two numbers	SOMETIMES It is but only if they are evenly spaced, i.e. there is the same difference between the numbers.
SOMETIMES Only if the difference between the two middle numbers is even.	SOMETIMES It would for example work with 1,3,3,5 but not with 2,10,10
SOMETIMES It is true for 1,1,2,2,2 but not for 2,3,33	NEVER It must be bigger than the smallest number and smaller than the biggest (unless all the numbers are the same)

Measures

M

Angles are between zero and 360 degrees	3 miles uphill is further than 3 miles downhill
If a has more digits than b, then a > b	If a number’s digits add up to 36, the number is in the 9x table
Distances measured in km are further than distances measured in cm	if a number’s digits add up to 6, the number is in the 3 times table

Measures

M

SOMETIMES More than a full turn gives angles greater than 360 degrees	NEVER They are the same distance (but one might feel further if you are walking!)
SOMETIMES 364 >99 but 3.64 < 99	ALWAYS This is known as a 'divisibility test' for multiples of 9; in fact it works if they add up to any multiple of 9.
SOMETIMES but 1km < 1.5 million cm	ALWAYS As 6 is double 3.

Prime Numbers

H

Prime numbers are odd	Whole non-primes greater than 2 can be made by multiplying two or more primes
Prime numbers over 3 are 1 away from a multiple of 6	Double a prime is an even number
The square of any prime bigger than 3 is 1 more than a multiple of 24	A prime selected at random is the largest prime number

Prime Numbers

H

SOMETIMES All of them are odd EXCEPT the first prime number, 2. This is because all other even numbers can't be prime as they have a factor of 2!	ALWAYS This is why primes are sometimes called the building blocks of number – you can literally build any non-prime number greater than 2 from them.
ALWAYS They can't be 2 or 4 away as they would be even. They can't be 3 away as they would be multiples of 3. If they are 5 away from a multiple of 6 they must be 1 away from the next one!	ALWAYS Double ANY whole number is even, whether it is prime or not.
ALWAYS A prime can either be (6n+1) or (6n-1). Square either of these and subtract 1 and you get an expression with a factor 12 and at least 1 other even factor.	NEVER Prime numbers go on for ever – there cannot be a highest one., as proved by Euclid in 300BC.

Chestnuts

M

The larger the coin, the larger the value	Adding 10 to an integer changes the units figure
Dividing makes numbers smaller	Rectangles can be cut into squares
Adding makes things bigger	Subtracting makes things smaller

Chestnuts

M

SOMETIMES **A 2p is larger than a 5p**	**SOMETIMES** **Only for numbers between −9 and −1**
SOMETIMES **Sometimes it does not. for example 0.5 divided by 0.5 is 1**	**SOMETIMES** **Not all can, but the ones that can are known as 'perfect rectangles'**
SOMETIMES **As long as you are adding a positive quantity**	**SOMETIMES** **As long as you are subtracting a positive quantity**

Lucky Dip (1)

H

The longer the perimeter the larger the area	Bar charts are the best way to show different amounts
Numbers in the 51 times table are prime	Between two whole numbers there is always another one
Between two fractions there is always another one	Between two decimals there is always another one

Lucky Dip (1)

H

SOMETIMES **Think about cutting a rectangle from the edge of a larger rectangle - longer perimeter, smaller area.**	**SOMETIMES** **They are useful for showing bigger and smaller amounts but not as good as pie charts for comparing quantities**
NEVER **They will all have a factor of 3 (and 17, and 51, and possibly others)**	**SOMETIMES** **Unless they are adjacent integers**
ALWAYS **Can be found, for example, by doubling both numerator and denominator and adding 1 to new numerator**	**ALWAYS** **There are an infinite number of decimal places so adding another decimal place to lower number with any non-zero digit will work.**

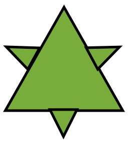
Lucky Dip (2)

H

Decimals can be expressed as a fraction	Fractions can be expressed as a decimal
A shape with exactly 3 lines of symmetry is an equilateral triangle	The factors of a number are smaller than its multiples
2 numbers that are not 1 have a product of 1	2 numbers that are not zero have a product of zero

Lucky Dip (2)

H

ALWAYS	SOMETIMES
Any decimal is a fraction over a power of 10 (10,100,1000 etc)	Those with a denominator of 3,6,7 or 9 cannot usually be expressed as a decimal as these are not factors of 10,100,100 etc.
SOMETIMES	**SOMETIMES**
There are other shapes with 3 lines– here is an example 	Or always if you exclude the number itself
SOMETIMES	**NEVER**
e.g. 0.25 and 4	This is why we can solve quadratics by factorising

ACKNOWLEDGEMENTS

Whoever first talked about standing on the shoulders of giants could well have been referring to this book.

I feel that apart from coming up with a few of my own new interesting ASN statements, I have built on the work of many others in order to compile this set of statements.

As I mentioned in the introduction, while searching the internet for a book of these statements, I discovered that amazingly no such book existed, though there were several books each containing a few excellent examples of the genre. This certainly helped me decide to write this book, but also made me realise that (somewhat irritatingly from my point of view) it was going to be nigh-on impossible to be certain where any particular statement originated, since several similar or identical statements were listed independently in lots of different websites and books.

I have used many of them either in their original or amended form, and am grateful. Even with the statements that I wrote specifically for this book, I found some very similar versions 'out there', and there are probably more than a few examples of genuine independent co-creation. I hope nobody will take offence (think Newton v. Liebnitz).

So, the best that I can hope to do here is simply to thank everyone who has inspired the book, either by their work on this area, or by the statements that they have generously made available on the internet for teachers to draw on.

AJ, January 2012

Below are the three main sources I have used during my research; you may well discover several others. My thanks to to colleagues who have sent me ideas or pointed me in the direction of a few good questions; whether they made it into the final edit or not, I am very grateful to you.

Enjoy!

National Centre for Excellence in the Teaching of Mathematics – www.ncetm.org

ATM bookshop – http://www.atm.org.uk/shop/bookshop.html

National Strategies website – now archived but still with limited access.

In memory of John Voller 1936 – 2012